The concept of λ-*central element* of an algebra, introduced by Vaggione in [1], generalize both central idempotent elements in rings with identity and neutral complemented elements in a bounded lattice. Essentially, they correspond to elements $(0,1)$ in direct products $A \times B$. The authors study varieties $\mathcal{V}$ having *equationally definable factor congruences (EDFC)*; this property states that there exists a conjunction of equations which defines the kernel of the projection $\pi_1 : A \times B \to A$ by using the associated central element as a parameter.

They prove that this condition is equivalent to definability by an open formula and they show that the kernel of the other projection may be defined by a universal first-order formula, regardless of the cardinality of the language of $\mathcal{V}$. Bounded semilattices and semidegenerate congruence-modular varieties have EDFC, and the authors provide refined versions of their results in those cases, and also in the congruence-permutable case. For the latter they prove that the class of directly indecomposables is axiomatizable by $\forall\exists$ formulas.

Finally, they prove that several of the first-order definitions they obtain are optimal in terms of quantifier alternation.

References

[1] D. Vaggione, $\mathcal{V}$ with factorable congruences and $\mathcal{V} = \mathbf{I}\mathbf{\Gamma}^a(\mathcal{V}_{DI})$ imply $\mathcal{V}$ is a discriminator variety, *Acta Sci. Math.* **62**: 359–368 (1996).