The present work refines previous results by Cabessa and other authors concerning the expressive power of *recurrent neural networks (RNN),* by looking at the topological complexity of the $\omega$-language accepted by them. The *deterministic* RNN model comprises input and output cells holding, at any time $t \in \mathbb{N}$, Boolean vectors $\vec{u} = (u_1, \ldots, u_P) \in \{0,1\}^P = \mathbb{B}^P$ and $\vec{y}$ (respectively) and inner sigmoid cells which hold a vector $\vec{x}$ with real values between 0 and 1. The dynamics of the RNN has the following form

$$\vec{x}(t+1) := \sigma(F_t(\vec{u}(t), \vec{x}(t)))$$
$$\vec{y}(t+1) := \chi_{[1,\infty)}(G_t(\vec{u}(t), \vec{x}(t))),$$

where $\chi$ is the indicator function, $\sigma$ is the "linear-sigmoid" with range $[0,1]$, and $F_t$ and $G_t$ are linear functions with coefficients depending on $t$. A sequence $\{\vec{u}(t)\}_{t\in\mathbb{N}}$ is *accepted* if the set of output states $\vec{y}(t)$ that appear infinitely often through the evolution belongs to a prescribed fixed family of sets. In a totally analogous manner to nondeterministic Turing machines, *nondeterministic* RNN are defined.

Cabessa and Villa [MR3521994] proved that if the $F_t$ and $G_t$ above do not depend on $t$ and have rational coefficients, the language accepted by the RNN is a Boolean combination of (lightface) $\Pi^0_2$ subsets of the Cantor space $(\mathbb{B}^P)^{\mathbb{N}}$. If we allow at least one coefficient $\alpha(t)$ depending on $t$ with values in $\{0,1\}$, or at least one irrational constant coefficient $r$, all Boolean combinations of $G_\delta$ subsets of $(\mathbb{B}^P)^{\mathbb{N}}$ are obtained. On the other hand, Cabessa and Duparc [MR3447446] studied nondeterministic RNN and obtained analogous results but with analytic subsets instead of Boolean combinations of $\Pi^0_2$.

In this work, the authors prove relative versions of the results of the previous paragraph, referring to the coefficient $\alpha \in \mathbb{B}^{\mathbb{N}}$ or the real $r$. For instance, $L \subseteq (\mathbb{B}^P)^{\mathbb{N}}$ belongs to $\Sigma^1_1(\alpha)$ if and only if it is accepted by a nondeterministic RNN with only $\alpha$ as the only coefficient depending on $t$, if and only if it is accepted by a nondeterministic RNN with constant coefficients and with the only (eventually) irrational coefficient being $r_\alpha := \sum_{i=1}^{\infty} \frac{2\alpha_i+1}{4^i}$.